Chapter 11 Series Part 2

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1. (a) The sum of the first two terms of a geometric progression is 10 and the third term is 9.

(i) Find the possible values of the common ratio and the first term.

[5]

$a + ar = 10$

$ar^2 = 9$

$a = \frac{9}{r^2}$

$\frac{9}{r^2} + \cancel{r} \times \frac{9}{r^{\cancel{2}}} = 10$

$\frac{9}{r^2} + \frac{9}{r} = 10$

$(\times r^2)$

$9 + 9r - 10r^2 = 0$

$(\times -1)$

$10r^2 - 9r - 9 = 0$

$r = \frac{3}{2}$ or $r = -\frac{3}{5}$

$a = \frac{9}{r^2}$ $\quad a = 25$

$= 4$

$-1 < r < 1$

(ii) Find the sum to infinity of the convergent progression.

[1]

$S_\infty = \frac{a}{1-r} = \frac{25}{1+\frac{3}{5}} = 15\frac{5}{8}$

(b) In an arithmetic progression, $u_1 = -10$ and $u_4 = 14$. Find $u_{100} + u_{101} + u_{102} + \ldots + u_{200}$, the sum of the 100th to the 200th terms of the progression.

[4]

$a = -10$

$a + 3d = 14$

$3d = 24$

$d = 8$

$a = u_{100} = a + 99d$

$= -10 + 99 \times 8$

$= 782$

$S_{101} = \frac{101}{2}\left((782) \times 2 + 100d\right)$

$= 119382$

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2. (a) An arithmetic progression has a second term of -14 and a sum to 21 terms of 84. Find the first term and the 21st term of this progression.

[5]

$a+d=-14$

$S_{21}=\frac{21}{2}(2a+20d)$

$84=21a+210d \quad \div 21$

$4=\cancel{a}+10d$

$\overset{+}{-}14=\overset{-}{\cancel{a}}\overset{-}{+}d$

$18=9d$

$d=2$

$a+d=-14$

$a+2=-14$

$a=-16$

$U_{21}=a+20d=-16+40$

$=24$

(b) A geometric progression has a second term of $27p^2$ and a fifth term of p^5. The common ratio, r, is such that $0 < r < 1$.

(i) Find r in terms of p. [2]

$ar = 27p^2$

$a = \frac{27p^2}{r}$

$ar^4 = p^5$

$\frac{27p^2}{\cancel{r}\,1} \times r^{\cancel{4}3} = p^5$

$27p^2 \times r^3 = p^5$

$r^3 = \frac{p^3}{27}$

$r = p/3$

(ii) Hence find, in terms of p, the sum to infinity of the progression. [3]

$S_\infty = \frac{a}{1-r} = \frac{27p^2}{p/3} \times \frac{1}{1-\frac{p}{3}}$

$= 81p \times \frac{1}{\frac{3-p}{3}} = 81p \times \frac{3}{3-p}$

$= \frac{243p}{3-p}$

(iii) Given that the sum to infinity is 81, find the value of p. [2]

$81 = \frac{243p}{3-p}$

$3-p = \frac{243p}{81}$

$3-p = 3p$

$3 = 4p$

$p = \frac{3}{4}$

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3.(a) An arithmetic progression has a first term of 7 and a common difference of 0.4. Find the least number of terms so that the sum of the progression is greater than 300.

[4]

$a = 7$

$d = 0.4$

$S_n > 300$

$\frac{n}{2}(2a + (n-1)d) > 300$

$n(14 + 0.4n - 0.4) > 600$

$n(13.6 + 0.4n) > 600$

$13.6n + 0.4n^2 - 600 > 0$

$n = 26$

$n = -59.3$

$n < -59.3$ or $n > 25.3$

(reject) $\quad n = 26$

(b) The sum of the first two terms of a geometric progression is 9 and its sum to infinity is 36. Given that the terms of the progression are positive, find the common ratio.

[4]

$a + ar = 9 \rightarrow a(1+r) = 9$

$a = \frac{9}{1+r}$

$\frac{a}{1-r} = 36$

$\frac{9}{1+r} \times \frac{1}{1-r} = 36$

$\frac{1}{1-r^2} = 4$

$1 - r^2 = \frac{1}{4}$

$r^2 = \frac{3}{4}$

$r = \frac{\sqrt{3}}{2}$

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4.(a) The first 5 terms of a sequence are given below.

4 -2 1 -0.5 0.25

(i) Find the 20th term of the sequence.

[2]

$a = 4$

$r = -\frac{1}{2}$

$U_{20} = ar^{19} = 4 \times \left(-\frac{1}{2}\right)^{19}$

$= -\frac{1}{131072}$

(ii) Explain why the sum to infinity exists for this sequence and find the value of this sum.

[2]

because $r = -\frac{1}{2}$ and so it lies between -1 & 1.

$S_\infty = \frac{a}{1-r} = \frac{4}{1+\frac{1}{2}} = 4 \times \frac{2}{3}$

$= \frac{8}{3} = 2\frac{2}{3}$

(b) The tenth term of an arithmetic progression is 15 times the second term. The sum of the first 6 terms of the progression is 87.

(i) Find the common difference of the progression. [4]

$u_{10} = 15u_2$

$a + 9d = 15(a + d)$

$a + 9d = 15a + 15d$

$0 = 14a + 6d$

$\div 2$

$0 = 7a + 3d$

$S_6 = 87$

$\frac{6}{2}(2a + 5d) = 87$

$2a + 5d = 29$

$\times 3$

$6a + \cancel{15d} = 87$

$- \quad 35a \mp \cancel{15d} \mp 0$

$-29a = 87$

$a = -3$

$2a + 5d = 29$

$-6 + 5d = 29$

$5d = 35$

$d = 7$

(ii)For this progression, the *n*th term is 6990. Find the value of *n*. [3]

n^{th} term $= a + (n-1)d$

$6990 = -3 + (n-1)7$

$6993 = (n-1)7$

$n - 1 = 999$

$n = 1000$

5.(a) An arithmetic progression has a second term of 8 and a fourth term of 18. Find the least number of terms for which the sum of this progression is greater than 1560.

[6]

$$\cancel{a}+d=8$$
$$-\ \cancel{a}+3d=18$$
$$-2d=-10$$
$$d=5$$

$$a+d=8$$
$$a=3$$

$$S_n > 1560$$
$$\frac{n}{2}(2a+(n-1)d) > 1560$$
$$\frac{n}{2}(6+(n-1)5) > 1560$$
$$n(6+5n-5) > 3120$$
$$n+5n^2-3120 > 0$$

$n=24.9$
$n=-25.1$

$n<-25.1$ or $n>24.9$
(reject)
$n=25$

(b) A geometric progression has a sum to infinity of 72. The sum of the first 3 terms of this progression is $\frac{333}{8}$.

(i) Find the value of the common ratio. [5]

$$S_\infty = 72$$

$$\frac{a}{1-r} = 72$$

$$a = 72 - 72r$$

$$\frac{a(1-r^n)}{1-r} = \frac{333}{8}$$

$$\frac{72\cancel{(1-r)} \times (1-r^3)}{\cancel{1-r}} = \frac{333}{8}$$

$$1-r^3 = \frac{37}{64}$$

$$-r^3 = -\frac{27}{64}$$

$$r^3 = \frac{27}{64}$$

$$r = \frac{3}{4}$$

(ii) Hence find the value of the first term. [1]

$$a = 72(1-r)$$

$$= 72 \times \frac{1}{4}$$

$$= 18$$

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6.The 7th and 10th terms of an arithmetic progression are 158 and 149 respectively.

(a) Find the common difference and the first term of the progression. [3]

$$\cancel{a}+6d=158$$
$$-\ \cancel{a}+9d=149$$
$$-3d=9$$
$$d=-3$$

$$a+6d=158$$
$$a-18=158$$
$$a=176$$

(b) Find the least number of terms of the progression for their sum to be negative. [3]

$$S_n<0$$

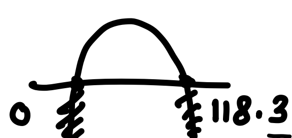
$$\frac{n}{2}(2a+(n-1)d)<0$$

$$\frac{n}{2}(2\times 176+(n-1)-3)<0$$

$$\frac{n}{2}(352-3n+3)<0$$

$$n(355-3n)<0 \qquad n=0$$
$$n=118.3$$

$$355n-3n^2<0$$



$n<0$ (reject) or $n>118.3$

$n=119$

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7.(a) The sum of the first 4 terms of an arithmetic progression is 38 and the sum of the next 4 terms is 86. Find the first term and the common difference.

[5]

$$S_4 = 38$$

$$2(2a + 3d) = 38$$

$$2a + 3d = 19 \quad — \quad 1$$

$$2(2 \times U_5 + 3d) = 86$$

$$2(a + 4d) + 3d = 43$$

$$2a + 8d + 3d = 43$$

$$2\cancel{a} + 11d = 43$$

$$- \quad \cancel{2}a + 3d = 19$$

$$8d = 24$$

$$d = 3$$

$$2a + 11d = 43$$

$$2a + 33 = 43$$

$$2a = 10$$

$$a = 5$$

(b) The third term of a geometric progression is 12 and the sixth term is -96. Find the sum of the first 10 terms of this progression.

[6]

$ar^2 = 12 \longrightarrow a = \frac{12}{r^2}$

$ar^5 = -96$

$\frac{12}{\cancel{r^2}} \times r^{\cancel{5}\,3} = -96$

$r^3 = -8$

$r = -2$

$a = \frac{12}{4} = 3$

$S_{10} = a\left(\frac{1-r^{10}}{1-r}\right)$

$= 3\left(\frac{1-(-2)^{10}}{1+2}\right) = -1023$